

EDUCATION Quick Reference Guide:

Students experience informal problem-solving situations throughout their everyday lives. For example, they may determine how much older they are than a friend, add points in a game, or organize and count the coins in a jar. Students formalize their problem-solving skills first through the application of addition and subtraction. They begin by modeling one-step, real-world problems and progress to solving two-step problems using equations and symbols. This guide elaborates on *Table 1: Common addition and subtractions* in the *Massachusetts Curriculum Framework for Mathematics*.

By learning operations in context, students develop numerical fluency and connect procedural knowledge to real-world situations. As Table 1 shows, the simple context 2 and 3 make 5 lends itself to multiple interpretations depending on the position of the unknown.

Grades Pre-K & K: Modeling and Solving Problems Within 10

Pre-kindergarten students begin their study of addition and subtraction situations by modeling real-world situations with concrete objects (PK.OA.A.1). By modeling real-world situations, students reinforce the conceptual foundations of addition and subtraction as operations while also abstracting real-world situations for the first time. Kindergartners continue to work with addition and subtraction situations within 10, specifically problems that involve *putting together, adding to, taking apart,* and *taking from* (K.OA.A). Kindergartners also model addition and subtraction situations using more complex methods, including drawings and equations.

Grades 1 & 2: One- and Two-Step Word Problems

Table 1 is directly referenced in standards for grades 1 and 2 (1.OA.A.1 and 2.OA.A.1). Students in both grades solve word problems involving all situations described in Table 1. In grade 1, students work with numbers within 20 to solve one-step word problems. The skills they acquire by addressing these standards are also applied to other standards, such as 1.OA.D.7, where they begin to interpret the meaning of the equal sign (=), and 1.OA.D.8, where they determine the unknown number in addition and subtraction problems. In grade 2, students progress to working with numbers within 100 to solve one- and two-step word problems. Second-grade students use comparison situations as they work with picture and bar graphs to represent data sets (2.MD.D.10).

Change Situations (Add To and Take From)

Change situations can involve three types of unknowns: *result, change*, and *start*. A distinctive feature of all change situations is that, as the name describes, something changes about the situation, often with the passage of time. Traditional addition and subtraction problems are *result unknown* situations however, students must also progress to solving *change unknown* and *start unknown* problems. Although the examples supplied in Table 1 are simple, students work with increasingly complex change situations throughout their academic careers.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5-2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3

Put Together and Take Apart Situations

Put together and *take apart* situations illustrate the composition of numbers and the relationships between addition and subtraction. Although they exhibit similarities to change situations, *put together* and *take apart* situations do not involve a change in a situation. Rather, these problems describe situations where some components are unknown and students must shift their thinking from the total to the addends. *Put together* and *take apart* situations call particular attention to the equal sign (=) as symbolizing equivalence rather than exclusively completing an operation.

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, $5 = 5 + 05 = 1 + 4$, $5 = 4 + 15 = 2 + 3$, $5 = 3 + 2$

Comparison Situations

Comparison problems marry addition and subtraction, asking students to determine the relative value of quantities within a situation. Students interpret the comparative language used to describe the situation: *who* or *what* has *more* or *less* than a referent quantity. Students also use this language as they describe their solutions in the context of the situation.

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	 ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ? 	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5-3=?, ?+3=5

Balanced Mathematical Instruction

To achieve mathematical understanding, students should be actively engaged in **meaningful mathematics**. The standards focus on developing students' conceptual understanding, procedural fluency, and problem-solving applications.

By encountering addition and subtraction problems in a context, students link their procedural understanding with real-world situations. As they work, students develop a deep conceptual understanding of number composition and decomposition, which is critical to understanding how and why algorithms work.



Check It Out!

Mathematics Learning in Early Childhood: Paths toward Excellence and Equity: <u>http://bit.ly/2AsmYa9</u> Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction: <u>http://bit.ly/2AZYeXO</u>