Quick Reference Guide: Standards for Mathematical Practice *Grades 6 - 8*

To become college and career ready, students must be able to problem-solve, reason and prove, communicate, represent, make connections, conceptualize, and strategize in mathematics. While content standards are specified by grade level, the Standards for Mathematical Practice evolve and mature over the years along with students' cognitive development. By integrating content and practice standards, students become practitioners of the discipline of mathematics. **This guide describes how mathematically proficient students in grades 6–8 might demonstrate the eight Standards for Mathematical Practice in the *Massachusetts Curriculum Framework for Mathematics.***

> "The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the habits of mind—used to create the results."
>
> *-Al Cuoco, Paul Goldenberg, & June Mark*

1. Make sense of problems and persevere in solving them.

Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. They analyze problem conditions and goals, translating verbal descriptions into mathematical expressions, equations, or drawings as part of the process. They consider analogous problems, and they try special cases and simpler forms of the original in order to gain insight into its solution.

Example: To understand why a 20% discount followed by a 20% markup does not return an item to its original price, a student might translate the situation into a tape diagram or a general equation; or they might first consider the result for an item priced at $1.00 or $10.00.

2. Reason abstractly and quantitatively.

Mathematically proficient middle school students make sense of quantities and relationships in problem situations. They represent problem situations using symbols, and then manipulate those symbols in search of a solution (decontextualize). They may pause during the problem-solving process to check for meaning or to use the units of measure in the problem to clarify possible next steps (contextualize).

Example: Students may apply ratio reasoning to convert measurement units and/or proportional relationships to solve percent problems. They might then explain their answer verbally in terms of the context.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They make and explore the validity of conjectures. They can recognize and appreciate the use of counterexamples to identify common errors in algebraic manipulation.

Example: Students might debate the validity of thinking that $5 - 2x$ is equivalent to $3x$ through the use of known definitions and mathematical properties.

4. Model with mathematics.

Mathematically proficient middle school students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a given relationship and represent situations using such tools as diagrams, tables, graphs, flowcharts, and formulas. They can analyze their representations mathematically, use the results in the context of the situation, and then reflect on whether the results make sense while possibly improving the model.

Example: Students might apply proportional reasoning to plan a school event or use a set of linear inequalities to analyze a problem in the community.

5. Use appropriate tools strategically.

Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem and while exploring a mathematical relationship. These tools might include pencil and paper, concrete models, a ruler, a protractor, a graphing calculator, a spreadsheet, a statistical package, or dynamic geometry software.

Examples: Students might use estimation to check reasonableness, graphs to model functions, algebra tiles to see how properties of operations apply to algebraic expressions, graphing calculators to solve systems of equations, and dynamic geometry software to discover properties of parallelograms.

6. Attend to precision.

Mathematically proficient middle school students communicate precisely to others both verbally and in writing. They present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, well-chosen details, and precise language.

Example: Students might use the definition of rational numbers to explain why a number is irrational. They might also describe congruence and similarity in terms of transformations in the plane.

7. Look for and make use of structure.

Mathematically proficient middle school students look closely to discern a pattern or structure. They can step back for an overview and shift perspective. This may involve viewing complicated elements as being constructed of single objects with their own contextual meanings.

Example: Students might describe their interpretation of the structure of an expression based on its individual elements—for instance, understanding $1.05a$ as an original value, a, plus 5% of that value, $0.05a$.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient middle school students notice if calculations are repeated, and look for both general methods and shortcuts. They continually evaluate the reasonableness of their intermediate and final results.

Example: Working with tables of equivalent ratios, students might deduce the corresponding multiplicative relationships and make generalizations about relationships to rates.